

QUESTION 2008

Group - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) If $A = \{1, 2, 3, 4, 8\}$, $B = \{2, 4, 6, 7\}$, then $A \Delta B$ is

a) $\{2, 4\}$

b) $\{1, 2, 3, 4, 6, 7, 8\}$

c) ϕ

d) $\{1, 3, 6, 7, 8\}$

ii) $\lim_{x \rightarrow \infty} (1+x)^{1/x}$ is equal to

a) 1

b) e

c) 0

d) ∞

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iii) $\frac{d}{dx}(\log_a x)$ is equal to

a) $\frac{1}{x}$

b) $\log(1/x)$

c) $(1/x)\log_a e$

d) $x \log e$

iv) If $y = \log x^2$, the value of $\frac{d^2 y}{dx^2}$ is

a) $\frac{2}{x^2}$

b) $-\frac{2}{x^2}$

c) $\frac{2}{x}$

d) $2x$

v) The matrix $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an

a) orthogonal matrix

b) idempotent matrix

c) identity matrix

d) none of these

vi) Derivative of x^4 with respect to x^2 is

a) $4x^3$

b) $2x^2$

c) $2x$

d) 4

vii) If the roots of the equation $ax^2 + bx + c = 0 (a \neq 0)$ are real and unequal, then its discriminant D satisfies

a) $D > 0$ and $D = a$ perfect square

b) $D = 0$

c) $D > 0$ and $D \neq a$ perfect square

d) $D < 0$

viii) If $A = \{1, 2, 3\}$, $B = \{2, 3, 6\}$, then $A \cup B$ is

a) $\{1, 2, 3\}$

b) $\{2, 3\}$

c) $\{1, 2, 3, 6\}$

d) none of these

ix) If α, β, γ be the roots of $x^3 - 3x^2 + 6x - 2 = 0$, then $\Sigma \alpha\beta$ is

a) 3

b) 6

c) 2

d) none of these

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x) If $f(x) = 3 + 2x$; when $x \geq 0$,
 $= -3 - 2x$; when $x < 0$,

then $\lim_{x \rightarrow \infty} f(x)$ is

a) 3

b) -3

c) 0

✓ d) none of these

xi) If $f(x) = \frac{x}{|x|}$; when $x \neq 0$

$= 1$; when $x = 0$, then

a) $f(x)$ is continuous at $x = 0$

b) $f(x)$ is continuous, but not differentiable at $x = 0$

✓ c) $f(x)$ is discontinuous at $x = 0$

d) none of these

xii) The value of $\int_{-1}^2 |x| dx$ is

a) 3

b) 5

✓ c) 5/2

c) none of these

Group - B

(Short Answer Type Questions)

2. If α, β, γ be the roots of the equation, $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma \alpha^3$.

See Topic: POLYNOMIAL, Short Answer Type Question No. 12.

3. If $u = \tan^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u$.

See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 3.

4. Prove the set of even integers (including zero) forms an additive group.

See Topic: BINARY COMPOSITION, Short Answer Type Question No. 4.

5. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 6.

6. If $P = \begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 5 \\ 7 & 12 \end{pmatrix}$, find the matrix R so that $5P + 3Q + 2R$ is a null matrix.

See Topic: MATRICES, Short Answer Type Question No. 8.

Group - C

(Long Answer Type Questions)

7. a) State Rolle's Theorem.

See Topic: EXPANSION OF FUNCTION, Short Answer Type Question No. 4.

b) Differentiate n times the following equation: $(1+x^2)y_2 + (2x-1)y_1 = 0$.

See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 8.

c) If show that = 0.

8. a) If p th, q th and r th terms of an A.P. are P , Q and R respectively, show that $p(Q - R) + q(R - P) + r(P - Q) = 0$.

See Topic: MISCELLANEOUS, Short Answer Type Question No. 3.

b) Show that the centroid of a triangle with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right).$$

See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 4.

c) Find the equation of a straight line through the point of intersection of lines $2x + 4 = 0$ and $3x + 4y - 5 = 0$ and that is perpendicular to the line $6x - 7y + 8 = 0$.

See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 5.

9. a) Show $0 < x < \frac{\pi}{2}$.

b) If $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0 \end{cases}$ then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$.

See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 6.

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c) Evaluate $\int \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$.

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 9.

10. a) Reduce the following equation to its canonical form and determine the nature of the conic represented by it: $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$.

b) Find the equation of the ellipse one of whose foci is $(-1, 1)$, eccentricity is 0.5 and the corresponding directrix is $y = x + 3$.

See Topic: GENERAL EQUATION OF SECOND DEGREE, Long Answer Type Question No. 3 & 4.

11. a) Solve the equation by Cardan's method, $2x^3 + 3x^2 + 3x + 1 = 0$.

b) Let $G = \{a \in R / -1 < a < 1\}$. Define the binary operation \otimes on G by $a \otimes b = \frac{a+b}{1+ab} \forall a, b \in G$.

Show that (G, \otimes) is a group.

c) Find the nature of the roots $x^4 + qx^2 + rx - s = 0$ by Descartes' rule of signs (where q, r, s , being positive).

a) See Topic: POLYNOMIAL, Long Answer Type Question No. 8.

b) See Topic: BINARY COMPOSITION, Long Answer Type Question No. 5.

c) See Topic: POLYNOMIAL, Short Answer Type Question No. 13.

12. a) If by a transformation of one rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$. Prove that $a^2 + b^2 = a'^2 + b'^2$.

b) Show that $\int_0^{\infty} \frac{dx}{(x+1)(x+2)} = \log 2$.

c) Use the method of integration to evaluate $\lim_{n \rightarrow \infty} \frac{1^k + 2^k + \dots + n^k}{n^{k+1}}; k > 0$.

a) See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 6.

b) See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 7.

c) See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 8.